Proof Theory and Mathematical Meaning of Paraconsistent C-Systems

Paolo Gentilini

September 2009

Abstract

The sequent versions $BC$, $CI$, $CIL$ of $bC$, $Ci$, $Cil$ presented in [Carnielli-Marcos 2002] are introduced and examined. In $CI$ and $CIL$ also a proper rule $RCi$ introducing the local consistency connective ($\cdot$)° is added. We show that $BC$, $CI$, $CIL$ admit of the cut elimination property and, in general, a weakened sub-formula property. On such results, an investigation on proof-theory and provability logic of $CI$-based arithmetical systems can be founded, in order to develop a constructive paraconsistent mathematics and a possible weakened Hilbert's program.

Moreover, a new arithmetical semantics is proposed for some paraconsistent C-systems, which are a relevant subclass of Logics of Formal Inconsistency (LFI) introduced by W. A. Carnielli et al [5]. Such semantics provides arithmetical models for the systems $BC$, $CI$, $CIL$ whose proof theory is introduced in the first part of the paper. Arithmetical semantics interprets C-system formulas into Provability Logic sentences of classical Arithmetic $PA$: thus, it links the notion of truth for a paraconsistent environment to the notion provability inside a classical environment. It makes true a peculiar infinite sub-class of contradictions $B \land \neg B$ and falsifies a sub-class of instances of the non contradiction principle $\neg (A \land \neg A)$. Moreover, Arithmetical models falsify both classical logic $LK$ and intuitionistic logic $LJ$, so that a kind of meta-logical completeness property of LFI-paraconsistent logic w.r.t. arithmetical semantics is proven. The possibility of interpreting $CI$-based paraconsistent Arithmetic into classical Provability Logic of $PA$ gives rise to interesting mathematical statements inside paraconsistent Arithmetic, as constructive conjectures and paradoxical assertions, whose investigation is a work in progress.

Key words


References


